Two combinatorial optimization problems coming from transportation

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CERMICS, Optimisation et Systèmes
First problem

Joint aircraft-crew planning and stochastic shortest paths
Objective: compute aircraft routes and crew routes minimizing costs

Graph $D = (V, A)$:

$V$ = flights

$A = (u, v)$ in $A$ if $v$ can follow $u$

$$
\min \sum_{p \in \mathcal{P}} c_p y_p \\
\text{s.t. } \sum_{p \ni v} y_p = 1 \quad v \in V \\
y_p \in \{0, 1\} \quad p \in \mathcal{P}$$

$$
\sum_{r \ni v} x_r = 1 \quad v \in V \\
x_r \in \{0, 1\} \quad r \in \mathcal{R} \\
\sum_{r \ni s} x_r \leq \sum_{p \ni s} y_p \quad s \in S$$
Algorithmic strategy

NP-hard problem, huge integer program: use of a mathheuristic.

Solve to optimality the crew pairing problem via column generation

\[
\begin{align*}
\min & \quad \sum_{p \in \mathcal{P}} c_p y_p \\
\text{s.t.} & \quad \sum_{p \ni v} y_p = 1 \quad v \in \mathcal{V} \\
& \quad y_p \in \{0, 1\} \quad p \in \mathcal{P}
\end{align*}
\]

A posteriori check existence of a compatible aircraft routing solution via integer program

\[
\begin{align*}
\sum_{r \ni v} x_r &= 1 \quad v \in \mathcal{V} \\
\sum_{r \ni s} x_r &\leq \sum_{p \ni s} y_p \quad s \in \mathcal{S}, \ p \ni s \\
x_r &\in \{0, 1\} \quad r \in \mathcal{R}
\end{align*}
\]

If not, solve again crew pairing via additional constraint

\[
\begin{align*}
z_s &\geq y_p \quad s \in \mathcal{S}, \ p \ni s \\
\sum_{s \in \mathcal{F}} z_s &\leq |\mathcal{F}| - 1
\end{align*}
\]
Control delay: working with \( p \in \mathcal{P} \) such that

\[
P(\text{delay along } p > \tau) < \varepsilon
\]

“In 1 − \( \varepsilon \) of the cases, the delay is smaller than \( \tau \) minutes.”

**Stochastic Resource Constraint Shortest Path** as a subproblem:

**Input.** Graph \( D = (V, A) \), two vertices \( s \) and \( t \), independent random travel times \( (X_a)_{a \in A} \), costs \( (c_a)_{a \in A} \).

**Output.** \( s-t \) path \( P \) with \( P(\sum_{a \in P} X_a > \tau) < \varepsilon \) and with minimum \( \sum_{a \in P} c_a \).
Algorithm for Stochastic Resource Constraint Shortest Path


Use stochastic lower bounds to discard uninteresting partial path.

Stochastic lower bound computed via a fixed-point equation:

\[
\begin{align*}
Z_t &= 0 \\
Z_v &= \bigwedge_{u \in N^+(v)} (X_{(v,u)} + Z_u) \quad v \in V
\end{align*}
\]
Experiments

Overall method currently implemented at AIR FRANCE.

Algorithm for **STOCHASTIC RESOURCE CONSTRAINT SHORTEST PATH able**

- to solve instances with 1600 vertices and 6500 arcs in less than 15 seconds.
- to replace $\mathbb{P}(\cdot > \tau)$ by other risk measures (such as CVaR).
Second problem

Shuttle scheduling
Increase the capacity of the Chunnel

PhD thesis LAURENT DAUDET 2014-2017, Chaire EUROTUNNEL

Trains in the tunnel: Eurostars, freight trains, passengers shuttles (PAX), freight shuttles (HGV)

First objective: Increase capacity in HGV’s
Second objective: Increase capacity in PAX’s

Constraints: Safety, “equity” schedule of Eurostars, fixed number of Eurostars and freight trains, fixed number of departures
Improvement?

**EUROTUNNEL** current rules: cyclic schedule, (one hour period), discretized (1 minute), 1 Eurostar every 30 minutes.

Possible way to improve...

- “Relax” these rules to improve capacity.
- Decrease waiting time of passengers.
There is room for optimization

Currently: 10 shuttles per hour, in each direction.

Elementary experiments with integer programming (cPLEX) show

- with arbitrarily small discretization: 13 shuttles per hour
- with 1 Eurostar every 25-35 minutes: 11 shuttles per hour
- with both + 2 hour cycle: 13.5 shuttles per hour.
Decrease waiting time: simplified version

- One type of mobile, one direction, known cumulated demand $D(t)$ for all $t \in [0, T]$.

- Objective: minimize maximum waiting time of passengers.

- Constraints:
  - Loading: constant rate $\rho$, starts at the arrival last passenger of shuttle.
  - Maximal load ($C$) of shuttles, fixed number of departures ($N$).
  - Safety constraints, speed limit.

**Theorem**

*Problem polynomially solvable.*
A more realistic problem

- One type of mobile, two directions, known cumulated demand $D(t)$ for all $t \in [0, T]$.

- Objective: minimize maximum waiting time of passengers.

- Constraints:
  - Loading: constant rate $\rho$, starts at the arrival last passenger of shuttle.
  - Maximal load ($C$) of shuttles, fixed number of departures ($N$).
  - Safety constraints, speed limit.
Proposed approach

Lagrangean heuristic.

1. **remove the constraints** with $D(t)$ and add them to the objective function with a penalization
2. **solve this problem** (polynomial $\simeq$ simplified version)
3. **repeat** with new penalization $\rightarrow$ convergence to a good lower bound
4. **heuristically build a feasible solution** from the lower bound

Experiments currently carried out.
Merci.